

# Following the density perturbations through a bounce with AdS/CFT Correspondence

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**ABSTRACT:** The recently proposed coupled scalar tachyon bounce (CSTB) model is a bounce universe model based on Type IIB string theory. We investigate the dynamics of fluctuations across the bounce point and check whether the scale invariance of the spectrum of the primordial density perturbations generated during the phase of matter-dominated contraction is preserved by the bounce. To this end we utilize the AdS/CFT correspondence: we map the fluctuations onto the boundary before the onset of the strongly coupled gravitational interactions in the bulk. We endow time dependence to the bulk spacetime metric and find an exact solution to the dilaton equation in Type IIB string in an  $AdS_5 \times S^5$  background. The time-dependent dilaton determines dynamics of the gauge fields on the boundary. We can thus map the density fluctuations to the boundary and observe their evolution on the boundary even when the bulk gravity becomes strongly coupled near the bounce point. Allowing sufficient time after the bounce point as the bulk returns to weakly coupled state, we map the boundary fluctuations back and compare the post-bounce spectrum with the pre-bounce one. The scale invariance as well as the stability of the spectrum of the primordial density perturbations obtained in the contraction phase of the CST bounce universe model is proven to be unaffected by the bounce.

**KEYWORDS:** AdS/CFT, holography, bounce universe, dilaton dynamics

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## 1 Introduction

A universe with a bounce process (see for example [1, 2] for two most updated reviews) is a possible solution of the cosmic singularity problem [3, 4] in the standard cosmology within the inflation paradigm [5, 6]. The bounce universe postulates that a phase of matter-dominated contraction precedes the big bang during which the scale factor of the universe reaches a non-zero minimal value. There have been many attempts to extend the standard cosmology beyond the Big Bang, the most notable first effort being the Pre-Big-Bang cosmology [7, 8], and then the Ekpyrotic cosmology [9]. A breakthrough was due to the key observation by D. Wands [10] in which he pointed out a scale invariant spectrum of primordial density perturbations can be generated during a matter dominated contraction. Although the spectrum generated in his naive model was later proved to be unstable, it opens a new chapter in cosmological modeling of the early universe.

Building on many pioneering works to utilize AdS/CFT correspondence [11] in cosmological studies [12–39], in this paper, we use the correspondence to study how a spectrum generated during the contraction phase can evolve through the bounce in a particular bounce universe model. We are going to conduct our investigations on the

coupled scalar tachyon bounce (CSTB) model [40] constructed earlier, which is based on the D-brane and anti-D-brane dynamics in Type IIB string theory.

The CSTB model has been shown to solve the singularity, horizon and flatness problem [41]; it can produce a scale invariant as well as stable spectrum of primordial density perturbations [42, 43]. Furthermore predictions testable using dark matter direct detections have been extracted (for a wide class of bounce models) [44–47]. An out-of-thermal-equilibrium dynamics of matter production in the the bounce universe makes the bounce scenario very distinct [44] from the standard model of cosmology in which thermal equilibrium dynamics washes out early universe information. A short review of the key ideas can be found [45, 48]. We would like to further corroborate our model by investigating the fluctuations across the bounce.

The fact that CSTB is a string-inspired model and the bounce point may be strongly gravitationally-coupled prompts us to use the AdS/CFT correspondence [11] to study the evolution of the primordial density fluctuations in a Type IIB string background. We take the bulk spacetime metric to be a time dependent  $AdS_5 \times S^5$  with its four dimensional part being a FLRW (Friedmann-Lematre-Robertson-Walker) space-time. In [49] a recipe is provided to map the bulk dynamic, fro and back, to the boundary. In this work we improve on their recipe by finding a solution to the dilaton dynamic equation of motion with more realistic Type IIB fields configurations.

According to the AdS/CFT correspondence, which is a strong/weak duality, i.e. when the bulk fields are strongly coupled the boundary is described by a weakly coupled field theory, and vice versa, the bulk fields have dual-operators prescribed by the boundary theory. The dilaton field is related to the square of the gauge field strength, and the gauge coupling of the boundary theory is determined by the vev of the dilaton  $\phi$ . Therefore the first step is to find a time dependent solution of dilaton equation which, in turn, determines the dynamics of gauge fields on the boundary. Consequently when the boundary gauge field theory becomes weakly coupled during the contraction, we can map the bulk fluctuations onto the boundary and observe its evolution through the bounce.

The bounce process in the bulk could be potentially violent or highly singular in nature – although this is not the case for the CSTB model which enjoys a string theoretical completion at high energy and has a minimum radius – the gauge fields on the boundary, however, evolves most smoothly.

After the bounce, we map the evolved fluctuations – using again the AdS/CFT dictionary – back to the bulk as the gravitational dynamics return to a weakly coupled

state. The operation described above hence allows comparing the post-bounce spectrum with the pre-bounce spectrum and checking whether the scale invariance of the spectrum is respected by the bounce process.

The paper is organized as follows. In section 2 we present a time dependent dilaton solution with nonzero Ramond-Ramond charges in Type IIB string theory. We describe the cosmic background in which CSTB model can be constructed. In section 3, we use the results of the previous section to solve the equation of motion of the boundary gauge fields near bounce point, and match the solutions at different evolutionary phases; and finally check whether the spectrum is altered during the bounce. In section 4, we summarize our findings, discuss a potential caveat and remedies. We conclude with outlook on further studies with alternative solutions.

## 2 A time dependent dilaton solution to Type IIB supergravity

First of all we would like to find a solution of the dilaton in Type IIB supergravity with nonzero Ramond-Ramond potentials [50]. The CSTB cosmos is a string cosmological model that can be embedded into an exact string background with appropriate time dependence. The time dependence is necessary for cosmological studies. Altogether we need to generalize the AdS/CFT correspondence to incorporate time dependence in order to study how the spectrum of primordial density perturbations, generated before the bounce, is affected by the bounce dynamics.

The low energy effective theory of Type IIB string is given by [51]:

$$\begin{aligned}
S_{IIB} &= S_{NS} + S_R + S_{CS} \\
S_{NS} &= \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left( R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{12} |H_3|^2 \right) \\
S_R &= -\frac{1}{4\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left( |F_1|^2 + \frac{1}{3!} |\tilde{F}_3|^2 + \frac{1}{2 \times 5!} |\tilde{F}_5|^2 \right) \\
S_{CS} &= -\frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3
\end{aligned} \tag{2.1}$$

where the field strengths are defined as  $\tilde{F}_3 = F_3 - C_0 \wedge H_3$ ,  $\tilde{F}_5 = F_5 - \frac{1}{2} C_2 \wedge H_3 + \frac{1}{2} B_2 \wedge F_3$ , and  $F_3 = dC_2$ ,  $F_5 = dC_4$ ,  $H_3 = dB_2$ . The p-forms fields arise from the Ramond-Ramond sector and couple to D-branes of various dimensions; whereas  $\phi$  is the dilaton field we

are interested in. Note that there is an added constraint which should be imposed on the solution that the 5-form field strength:  $\tilde{F}_5$  is self-dual,  $\tilde{F}_5 = *\tilde{F}_5$ . The field equations derived from the action (2.1) should be consistent with, but do not imply, it.

The deformed  $AdS_5 \times S^5$  spacetime metric we will be working on is,

$$ds^2 = \frac{L^2}{z^2} [-dt^2 + a^2(t)\delta_{ij}dx^i dx^j + dz^2] + L^2 d\Omega_5^2 \quad (2.2)$$

where  $d\Omega_5^2$  being the metric of the unit  $S^5$  and  $a(t)$  being the scale factor of the 4-dimensional FLRW universe and  $L$  the AdS radius.

The equation of motions are [52]:

$$R_{\mu\nu} + 2\partial_\mu\partial_\nu\phi - \frac{1}{4}(H_3^2)_{\mu\nu} = e^{2\phi} \left[ \frac{1}{2}(F_1^2)_{\mu\nu} + \frac{1}{4}(\tilde{F}_3^2)_{\mu\nu} + \frac{1}{96}(\tilde{F}_5^2)_{\mu\nu} \right] - \frac{1}{4}g_{\mu\nu} \left( F_1^2 + \frac{1}{6}\tilde{F}_3^2 + \frac{1}{240}\tilde{F}_5^2 \right) \quad (2.3)$$

$$R - 4\partial_\mu\phi\partial^\mu\phi + 4\partial_\mu\partial^\mu\phi - \frac{1}{12}H^2 = 0 \quad (2.4)$$

$$*\tilde{F}_3 \wedge H_3 + d* dC_0 = 0 \quad (2.5)$$

$$2d*\tilde{F}_3 + H_3 \wedge \tilde{F}_5 + \frac{1}{2}B_2 \wedge d\tilde{F}_5 - dC_4 \wedge H_3 = 0 \quad (2.6)$$

$$d*\tilde{F}_5 = H_3 \wedge F_3 \quad (2.7)$$

$$-2d(e^{-2\phi}*H) + 2d(C_0*\tilde{F}_3) + dC_2 \wedge \tilde{F}_5 + \frac{1}{2}C_2 \wedge d\tilde{F}_5 - dC_4 \wedge dC_2 = 0 \quad (2.8)$$

In the above  $\mu, \nu = 0, 1 \dots 10$ ; and the subscripts,  $p$ , denote the ranks of p-form fields.

We need to make some sensible assumptions to solve this formidable array of equations. A common formula for the self-dual  $\tilde{F}_5$  is [53]:

$$\begin{aligned} \tilde{F}_5 = & r(\sqrt{-g_{00}g_{11}g_{22}g_{33}g_{44}}dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^4 \\ & - \sqrt{g_{55}g_{66}g_{77}g_{88}g_{99}}dx^5 \wedge dx^6 \wedge dx^7 \wedge dx^8 \wedge dx^9) , \end{aligned} \quad (2.9)$$

as we would like  $r$  to be a constant. Note that  $\tilde{F}_5 = dC_4 - \frac{1}{2}C_2 \wedge dB_2 + \frac{1}{2}B_2 \wedge dC_2$ , we can assume that  $B_2$  and  $C_2$  live on the  $AdS_5$  part and  $dC_4$  lives on the  $S^5$  part.

In the orthonormal basis, we can express them as:

$$B_2 = f_1 dy^0 \wedge dy^i + f_2 dy^i \wedge dy^j + f_3 dy^i \wedge dy^4 + f_4 dy^0 \wedge dy^4 , \quad (2.10)$$

$$C_2 = g_1 dy^0 \wedge dy^i + g_2 dy^i \wedge dy^j + g_3 dy^i \wedge dy^4 + g_4 dy^0 \wedge dy^4 , \quad (2.11)$$

where  $i = 1, 2, 3$ ,  $\{dy^\mu\}$  are the orthonormal basis, i.e.  $dy^\mu = \sqrt{g_{\mu\mu}} dx^\mu$ . To lessen the influence and difficulty caused by forms we assume that the coefficients  $f_1 \dots, g_1 \dots$  are at most linear in  $y^0$  and  $y^4$ , then we can get the expression for the  $AdS_5$  part of  $\tilde{F}_5$ :

$$\begin{aligned} \frac{1}{2}(B_2 \wedge dC_2 - C_2 \wedge dB_2) = & \frac{3}{2}[f_1 \frac{\partial g_2}{\partial y^4} + f_3 \frac{\partial g_2}{\partial y^0} + f_2(\frac{\partial g_1}{\partial y^4} + \frac{\partial g_3}{\partial y^0}) \\ & - g_1 \frac{\partial f_2}{\partial y^4} - g_3 \frac{\partial f_2}{\partial y^0} - g_2(\frac{\partial f_1}{\partial y^4} + \frac{\partial f_3}{\partial y^0})] . \end{aligned} \quad (2.12)$$

We will take these  $f_i$  to be constant and  $g_j$  to be linear in  $y^0$  and  $y^4$ , then the constant,  $r$ , mentioned above becomes:

$$r = \frac{3}{2}(f_1 h_3 + f_3 h_2 + f_2 h_1) \quad (2.13)$$

where  $h_1 = \frac{\partial g_1}{\partial y^4} + \frac{\partial g_3}{\partial y^0}$ ,  $h_2 = \frac{\partial g_2}{\partial y^0}$  and  $h_3 = \frac{\partial g_2}{\partial y^4}$ . Since  $f_4$  and  $g_4$  won't appear in the equations of forms, we'll take them to be zero. Therefore

$$H_3 = dB_2 = 0 \quad (2.14)$$

$$dC_2 = h_1 dy^0 \wedge dy^i \wedge dy^4 + h_2 dy^0 \wedge dy^i \wedge dy^j + h_3 dy^i \wedge dy^j \wedge dy^4 \quad (2.15)$$

$$dC_4 = -r dy^5 \wedge dy^6 \wedge dy^7 \wedge dy^8 \wedge dy^9 . \quad (2.16)$$

Putting these expressions of forms into equations (2.5) to (2.8) we arrives at

$$\frac{\partial C_0}{\partial y^4} = -r \frac{h_3}{h_1} \quad (2.17)$$

$$\frac{\partial C_0}{\partial y^0} = -r \frac{h_2}{h_1} \quad (2.18)$$

$$h_1^2 = h_2^2 - h_3^2 \quad (2.19)$$

Note here we consider the axion field  $C_0$  to be linear in time,  $y^0$ , and in,  $y^4$ , the spatial direction transverse to our 4-dimensional universe inside the  $AdS_5$ . So far what we do is to represent the forms by the coefficients  $f_i$  and  $h_j$ . In addition, we solve for (2.4) which is the Euler-Lagrange equation of  $\phi$ :

$$2\partial_\mu \partial_\nu \phi = 4\partial_\mu \phi \partial_\nu \phi - \frac{1}{2}g_{\mu\nu}(R + 4\partial_\rho \phi \partial^\rho \phi) . \quad (2.20)$$

Putting Equations (2.17) to (2.20) into (2.3), we get the equations of  $\phi$  when  $\mu\nu = 00, ii, 44$  (with the metric (2.2)):

$$\frac{3\dot{a}^2}{a^2} - \frac{6}{z^2} + 2\dot{\phi}^2 + 2\phi_{,z}^2 + \frac{6}{a^2}\phi_{,i}^2 = e^{2\phi} \frac{L^2}{z^2} \left( \frac{r^2 h_2^2}{2h_1^2} - 3h_1^2 - 3h_2^2 + \frac{9}{2}h_3^2 \right) \quad (2.21)$$

$$\frac{6a^2}{z^2} - 2a\ddot{a} - \dot{a}^2 + 2a^2\dot{\phi}^2 - 2a^2\phi_{,z}^2 - 2\phi_{,i}^2 = e^{2\phi} \frac{a^2 L^2}{z^2} \frac{15h_1^2}{2} \quad (2.22)$$

$$\frac{6}{z^2} - \frac{3\ddot{a}}{a} - \frac{3\dot{a}^2}{a^2} + 2\dot{\phi}^2 + 2\phi_{,z}^2 - \frac{6}{a^2}\phi_{,i}^2 = e^{2\phi} \frac{L^2}{z^2} \left( \frac{r^2 h_3^2}{2h_1^2} + 3h_1^2 + \frac{9}{2}h_2^2 - 3h_3^2 \right) \quad (2.23)$$

These are quadratic first-order partial differential equations of  $\phi$ . Normally they are hard to solve, however, if we view them as linear equations of  $\dot{\phi}^2$ ,  $\phi_{,z}^2$  and  $\phi_{,i}^2$ , life becomes much easier:

$$\dot{\phi}^2 = \frac{1}{4} e^{2\phi} \frac{L^2}{z^2} \left( \frac{r^2 h_2^2}{3h_1^2} + \frac{r^2 h_3^2}{6h_1^2} + \frac{13}{2}h_1^2 - \frac{1}{2}h_2^2 + 2h_3^2 \right) + \frac{3\ddot{a}}{4a} - \frac{1}{z^2} \quad (2.24)$$

$$\frac{2\phi_{,i}^2}{a^2} = \frac{1}{6} \left[ e^{2\phi} \frac{L^2}{z^2} \left( \frac{r^2}{2} - \frac{27}{2}h_1^2 \right) + \frac{12}{z^2} - \frac{6\dot{a}^2}{a^2} - \frac{3\ddot{a}}{a} \right] \quad (2.25)$$

$$\phi_{,z}^2 = \frac{1}{4} e^{2\phi} \frac{L^2}{z^2} \left( \frac{r^2 h_2^2}{6h_1^2} + \frac{r^2 h_3^2}{3h_1^2} - \frac{13}{2}h_1^2 + 2h_2^2 - \frac{1}{2}h_3^2 \right) + \frac{1}{z^2} . \quad (2.26)$$

We would like  $\phi$  to be spatially homogeneous, i.e.  $\phi_{,i}^2 = 0$ ; we can take such an approximation of  $e^{2\phi}$  that the right side of equation (2.25) equals to zero, then

$$e^{2\phi} \frac{L^2}{z^2} = \left( \frac{6\dot{a}^2}{a^2} + \frac{3\ddot{a}}{a} - \frac{12}{z^2} \right) \left( \frac{r^2}{2} - \frac{27}{2}h_1^2 \right)^{-1} \quad (2.27)$$

Substituting it into (2.24) we obtain

$$\dot{\phi} = \frac{1}{2} \sqrt{\frac{6m\dot{a}^2}{a^2} + \frac{3(m+1)\ddot{a}}{a} - \frac{12m+4}{z^2}} \quad (2.28)$$

with  $m = \left( \frac{r^2}{3} + \frac{r^2 h_3^2}{2h_1^2} + \frac{3}{2}h_1^2 + \frac{9}{2}h_2^2 - 3h_3^2 \right) \left( \frac{r^2}{2} - \frac{27}{2}h_1^2 \right)^{-1}$ . The constant captures the effects of form fields  $C_2$ ,  $B_2$ ,  $C_0$  on the dilaton,  $\phi$ . In the next section we will see that it is  $\dot{\phi}$  that matters. Note that we should not solve (2.27) directly since it is actually a result of an approximation instead of an exact solution. If we want exact solutions to Equations (2.24) to (2.26) then the second partial derivatives of  $\phi$  should satisfy a constraint equation,  $\frac{\partial \dot{\phi}}{\partial z} = \frac{\partial \phi_{,z}}{\partial t}$ .

### 3 The evolution of the gauge-field fluctuations

The boundary gauge theory is described by  $\mathcal{N} = 4$  SYM theory, we will follow the notations in [49]. The Yang-Mills coupling is determined by the dilaton by  $g_{\text{YM}}^2 = e^\phi$ . The boundary theory is strongly coupled in the far past. As the universe contracts, the bulk gravity theory becomes more and more strongly coupled. Before we approach the bounce point, we map the fluctuations onto the boundary as it becomes weakly coupled at this point. We let the gauge field evolve well after the bounce ends and the bulk returns to a weakly coupled state.

After rescaling and gauge fixing, the equations of motion for the Fourier modes of the gauge fields  $\tilde{A}$  becomes [30]:

$$\ddot{\tilde{A}}_k + (k^2 + M_{\text{YM}}^2)\tilde{A}_k = 0 \quad (3.1)$$

where

$$M_{\text{YM}}^2 = \frac{\ddot{\phi}}{2} - \frac{\dot{\phi}^2}{4} . \quad (3.2)$$

Let us now zoom into the cosmic dynamics near the bounce point and consider the three phases of universe evolution in the CSTB model [40]:

$$\text{Deflation : } a = e^{-Ht}, t_1 < t < -t_f \quad (3.3)$$

$$\text{Smooth bounce : } a = \cosh(Ht), -t_1 \leq t \leq t_1 \quad (3.4)$$

$$\text{Inflation : } a = e^{Ht}, t_1 < t < t_f; \quad (3.5)$$

where  $t_1$  is the time when inflation starts and  $t_f$  is when it ends. The bounce process is symmetric about  $t = 0$ . The mapping happens at deflation and inflation phases while the bulk becomes strongly coupled. We solve the equations of motion in each phases:

1. Deflation:

Putting (3.3) and (2.28) into (3.2) we arrive at

$$M_{\text{YM}}^2 = -\frac{3}{16}(3m+1)H^2 + \frac{3m+1}{4z^2} \equiv M . \quad (3.6)$$

In (3.6) all the terms are effectively constant, we denote it as  $M$ . Putting it into (3.1) yields

$$\tilde{A}_k = D_1(k)e^{\beta t} + D_2(k)e^{-\beta t} \quad (3.7)$$

where  $\beta \equiv \sqrt{-k^2 - M}$ .



2. Smooth bounce:

Taking the first order of  $t$  we obtain

$$M_{\text{YM}}^2 = \frac{3mH^4t}{\sqrt{-\frac{12m+4}{z^2} + 3(m+1)H^2}} - \frac{1}{4} \left( -\frac{3m+1}{z^2} + \frac{3}{4}(m+1)H^2 \right) \equiv Pt + Q \quad (3.8)$$

which yields

$$\tilde{A}_k = E_1(k) \text{Ai} \left[ \frac{-k^2 - Q - Pt}{(-P)^{\frac{2}{3}}} \right] + E_2(k) \text{Bi} \left[ \frac{-k^2 - Q - Pt}{(-P)^{\frac{2}{3}}} \right] . \quad (3.9)$$

3. Inflation: In this case everything is same as deflation except the value of  $t$ . Therefore

$$\tilde{A}_k = F_1(k)e^{\beta t} + F_2(k)e^{-\beta t} . \quad (3.10)$$

We denote  $\pm t_0$  as the time of mapping and for the sake of convenience, we set the two modes of  $\tilde{A}_k$  to have the same amplitudes after the first mapping, i.e.

$$D_1(k)e^{-\beta t_0} = D_2(k)e^{\beta t_0} \quad (3.11)$$

We assume the arguments of both Airy functions to be small and that the  $(-P)^{\frac{1}{3}}t$  term dominates. Then we can asymptotically expand the Airy functions to first power in  $q \equiv \frac{-k^2 - Q - Pt}{(-P)^{\frac{2}{3}}}$ :

$$E_1(k) \text{Ai}(q) = \frac{\left(\frac{1}{3}\right)^{\frac{2}{3}}}{\Gamma\left(\frac{2}{3}\right)} E_1(k) \quad (3.12)$$

$$E_2(k) \text{Bi}(q) = \left[ \frac{\left(\frac{1}{3}\right)^{\frac{1}{6}}}{\Gamma\left(\frac{2}{3}\right)} + \frac{\left(\frac{1}{3}\right)^{\frac{5}{6}}}{\Gamma\left(\frac{4}{3}\right)} q \right] E_2(k) \quad (3.13)$$

Now we can match  $\tilde{A}_k$  and its derivative at the end of deflation and at the beginning of inflation, which we denote as  $-t_1$  and  $t_1$  respectively. Matching  $\tilde{A}_k$  yields:

$$D_1(k)e^{-\beta t_1} + D_2(k)e^{\beta t_1} = \frac{\left(\frac{1}{3}\right)^{\frac{2}{3}}}{\Gamma\left(\frac{2}{3}\right)} E_1(k) + \left[ \frac{\left(\frac{1}{3}\right)^{\frac{1}{6}}}{\Gamma\left(\frac{2}{3}\right)} + \frac{\left(\frac{1}{3}\right)^{\frac{5}{6}}}{\Gamma\left(\frac{4}{3}\right)} q_1 \right] E_2(k) \quad (3.14)$$

$$F_1(k)e^{\beta t_1} + F_2(k)e^{-\beta t_1} = \left[ \frac{\left(\frac{1}{3}\right)^{\frac{2}{3}}}{\Gamma\left(\frac{2}{3}\right)} + \frac{\left(\frac{1}{3}\right)^{\frac{4}{3}}}{\Gamma\left(\frac{4}{3}\right)} q_2 \right] E_1(k) + \left[ \frac{\left(\frac{1}{3}\right)^{\frac{1}{6}}}{\Gamma\left(\frac{2}{3}\right)} - \frac{\left(\frac{1}{3}\right)^{\frac{5}{6}}}{\Gamma\left(\frac{4}{3}\right)} q_2 \right] E_2(k) \quad (3.15)$$

where  $q_1 \equiv q(-t_1)$  and  $q_2 \equiv q(t_1)$ .

Matching  $\tilde{A}_k$  yields:

$$D_1(k)\beta e^{-\beta t_1} - D_2(k)\beta e^{\beta t_1} = \frac{\left(\frac{1}{3}\right)^{\frac{5}{6}}}{\Gamma\left(\frac{4}{3}\right)}(-P)^{\frac{1}{3}}E_2(k) \quad (3.16)$$

$$F_1(k)\beta e^{\beta t_1} - F_2(k)e^{-\beta t_1} = \frac{\left(\frac{1}{3}\right)^{\frac{4}{3}}}{\Gamma\left(\frac{4}{3}\right)}P^{\frac{1}{3}}E_1(k) + \frac{\left(\frac{1}{3}\right)^{\frac{5}{6}}}{\Gamma\left(\frac{4}{3}\right)}(-P)^{\frac{1}{3}}. \quad (3.17)$$

Solving Equations (3.14) to (3.17), we get

$$F_1(k) = \frac{1}{6\beta\Gamma\left(\frac{4}{3}\right)(-P)^{\frac{1}{3}}e^{2\beta t_1}} [D_1(k)I_1 + D_2(k)e^{2\beta t_1}I_2] \quad (3.18)$$

where

$$\begin{aligned} I_1 &= -3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(-P)^{\frac{2}{3}} + \beta(-P)^{\frac{1}{3}} \left[ 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(q_1 + q_2) + 9\Gamma\left(\frac{4}{3}\right) \right] \\ &\quad - \beta^2 \left[ 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)q_1q_2 + 3\Gamma\left(\frac{4}{3}\right)(q_1 + 2q_2) \right] \\ I_2 &= -3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(-P)^{\frac{2}{3}} + \beta(-P)^{\frac{1}{3}} \left[ -3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(q_1 + q_2) - 3\Gamma\left(\frac{4}{3}\right) \right] \\ &\quad - \beta^2 \left[ 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)q_1q_2 + 3\Gamma\left(\frac{4}{3}\right)(q_1 + 2q_2) \right], \end{aligned}$$

and

$$F_2(k) = \frac{1}{6\beta\Gamma\left(\frac{4}{3}\right)(-P)^{\frac{1}{3}}} [-D_1(k)J_1 + D_2(k)e^{2\beta t_1}J_2] \quad (3.19)$$

where

$$\begin{aligned} J_1 &= -3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(-P)^{\frac{2}{3}} + \beta(-P)^{\frac{1}{3}} \left[ 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(q_1 - q_2) + 3\Gamma\left(\frac{4}{3}\right) \right] \\ &\quad + \beta^2 \left[ 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)q_1q_2 + 3\Gamma\left(\frac{4}{3}\right)(q_1 + 2q_2) \right] \\ J_2 &= 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(-P)^{\frac{2}{3}} + \beta(-P)^{\frac{1}{3}} \left[ 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)(q_1 + q_2) + 9\Gamma\left(\frac{4}{3}\right) \right] \\ &\quad - \beta^2 \left[ 3^{\frac{1}{3}}\Gamma\left(\frac{2}{3}\right)q_1q_2 + 3\Gamma\left(\frac{4}{3}\right)(q_1 + 2q_2) \right] \end{aligned}$$

We are interested in small wave number limit compared to time scales above, i.e.  $kt_1 \ll 1$ . In the typical inflationary process  $Ht_f \sim 10^2$  and  $Ht_1 \sim 10^{-2}$ . Combining these two facts, we can assume

$$\beta = \sqrt{-k^2 - M} = \sqrt{-k^2 + \frac{3}{16}(3m+1)H^2 + \frac{3m+1}{4z^2}} \approx \sqrt{-M} \quad (3.20)$$

In addition,  $Q \sim M$ , we have

$$q_1 = \frac{-k^2 - Q + Pt_1}{(-P)^{\frac{2}{3}}} \approx \frac{-Q + Pt_1}{(-P)^{\frac{2}{3}}} . \quad (3.21)$$

Similar argument goes for  $q_2$  as well. From (3.20) and (3.21) we can see that  $I_1$ ,  $I_2$  and  $J_1$ ,  $J_2$  are independent of  $k$  when  $kt_1 \ll 1$ . From (3.11) we know

$$D_1(k) = \frac{1}{2} \tilde{A}_k(-t_0) e^{\beta t_0}, \quad (3.22)$$

$$D_2(k) = \frac{1}{2} \tilde{A}_k(-t_0) e^{-\beta t_0} . \quad (3.23)$$

Putting these two into (3.18) and (3.19), we obtain, after the second matching,  $\tilde{A}_k$ , has the form

$$\tilde{A}_k = (G_1 e^{\beta t} + G_2 e^{-\beta t}) \tilde{A}_k(-t_0) , \quad (3.24)$$

both  $G_1$  and  $G_2$  being independent of  $k$ . All in all we can conclude that after the bounce the spectral index is not altered.

The reconstruction of the bulk data from boundary data is elucidated in [49], we do not reproduce the arguments here. The punch line is that the  $k$ -dependence of the bulk fluctuations is completely determined by the  $k$ -dependence of the gauge field fluctuations,  $A_k(t)$ , which implies, in turn, that the evolution of the gauge fluctuations will preserve scale invariance across the bounce.

## 4 Conclusion and discussion

In this paper we used the AdS/CFT correspondence to show that, when  $kt_1 \ll 1$  or in the long-wave limits of the dual gauge fields on the boundary, the spectral index of the dilaton fluctuations is not altered as the universe described by the CSTB model undergoes a contraction prior to an expansion. The first step was to find a time-dependent solution of the dialton in a type IIB supergravity on a time-dependent

$AdS_5 \times S^5$ . We generalize the previous proposal of [49] in which a certain behavior of the dilaton  $\phi$  was assumed. We then utilize the dilaton solution to solve for the dynamics of gauge fields living on the boundary of the  $AdS_5$ . We study the gauge fields near the bounce point and matched their behavior at the transitional points in the different phases of cosmic evolution. The combined profile of gauge field evolution is smooth across the bounce point. The bounce process merely alters the amplitudes of the modes in the density perturbations, and it affects them in the same manner. Therefore it cannot alter the intrinsic scale dependence in the spectrum of matter perturbations generated during the phase of cosmic contraction prior to the bounce. Nevertheless as we can see from (3.18) and (3.19), as  $k$  becomes larger and larger, i.e. if we do not take long wavelength approximation, the dependence in  $k$  begins to show up in the spectrum, the implications of which are under investigation.

A clarifying remark is perhaps needed here to distinguish the two kinds of  $k$ -modes, and their time dependence, involved in the above discussion. The CST bounce universe undergoes a deflation, before the bounce point, accompanied by horizon crossings with modes with different  $k$ 's crossing at different times. This makes each  $k$ -mode in the primordial density perturbations pick up an implicit time dependence: only after this implicit time dependence is carefully taken into account can the spectrum has no overall time dependence. This is the key to the stability analysis on the spectrum generated from the CSTB model [42, 43]. But this commonly concerned  $k$ -dependence in the primordial spectrum is not what we have discussed so far in this work. The  $k$ -modes in (3.1) are the  $k$ -modes of the gauge fields living on the boundary of the  $AdS$ . They are involved in the mapping procedure and merely encode the bulk dynamics holographically at some particular points on the boundary. Therefore they cannot inject or remove any time dependence in the primordial spectrum. Once the dynamics are mapped onto the boundary, there is no more horizon crossing, the gauge fields evolve under their own equations of motion.

We have made several assumptions and approximations throughout the analysis. Different solutions of the dilaton could be obtained with different ansatz of the Ramond-Ramond field configurations. We have simply chosen the most manageable configuration yet retaining interesting physics. With higher orders of time dependence in the dilaton field we have to expand  $M_{YM}^2$  to the higher order in  $t$  when solving (3.1). A systematic study of the field configurations and the corresponding effects on the dilaton field is beyond the scope of this paper. These are nevertheless interesting effects together with higher  $\iota\alpha$  effects to the whole analysis, which we hope to address in a future publication.

Another line of research would be to properly set up and study the D-brane and anti-D-brane annihilation process for cosmological modeling. This is the basis for building early universe model from string theory. Going beyond effective field theory approach and beyond kinematic analysis or symmetry arguments can give a more realistic touch to string cosmology. What kind of string compactifications can give rise to a non-singular universe matching up to the array of precision cosmological observations should be the ultimate question to answer for string cosmologists.

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